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**Rapid Vaporization and Heating  
of Two Parallel  
Fuel Droplet Streams**

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0520-78 AT-22044 Abstract

An analysis is presented of the mechanisms of heat transfer and vaporization in an idealized two-dimensional fuel spray consisting of two parallel droplet streams injected in a hot gas. With the assumption of a constant relative velocity between the gas and liquid phases (inviscid approximation), a system of partial differential equations with linear operators and non-linear source terms is obtained. With the use of Green's functions, this system is transformed into a group of non-linear ordinary differential equations and integral equations which are readily solved with numerical techniques. The dimensionless groups influencing the problem are identified and a discussion is presented of the effects of four relevant parameters including: a vaporization-diffusion time ratio, a liquid-gas mass flow ratio, a Spalding transfer number, and a Reynolds number. The method yields qualitatively correct estimates of the behavior of the gas and liquid phases which may serve as guidelines in the development of costlier numerical schemes. The method can be further exploited to include chemical reactions and to predict ignition of parallel droplet streams.

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## Introduction

The high level of complexity and analytically untractable non-linearities present in the study of multidimensional, steady and unsteady spray models have forced the combustion scientist, particularly in recent years, to rely heavily on costly, large-scale, computational techniques which are able to handle the rather challenging amount of mathematical difficulties. Examples of these computational efforts have appeared recently in the literature<sup>1-4</sup>.

To aid in the understanding of the fundamentals of these phenomena, it is appropriate to develop, simultaneously with the numerical schemes, a less costly analytical approach based on simplified models that retain the essential physics of the problem. A basis for this approach has been shown in the linear model of Sirignano<sup>5</sup>.

In this work we establish the fundamentals of a multidimensional spray model aimed eventually at the study of the ignition of a stream of droplets by another stream which has already ignited.

We consider a two-dimensional uniform velocity gas flow in which the vaporizing droplets are represented by two continuous sheets traveling at a different velocity within the gas. Uniform heating and vaporization of the droplets is considered as well as gas diffusion in the transverse direction. Axial diffusion, momentum exchange, density variations and the effect of convection in the transverse direction are neglected. An improvement of the model to account for spatial variations in the droplet temperatures can be achieved with the use of eigen-function expansions and integral equations<sup>6-7</sup>. The chemical reaction terms necessary for the study of the parallel-stream ignition mechanisms can be incorporated in the present model at some future point. Realize that with the use of Shvab-Zeldovich variables and a one-step

chemical reaction, all but one of the governing equations in the reacting case can be cast into a non-reactive form.

### Analysis

In this work we consider a two-dimensional, constant property gas which flows in steady state between two planes of constant but different temperature. Two droplet sheets parallel to the boundary planes are flowing within the gas with the same constant velocity  $U_d$  which is different from the gas velocity,  $U_g$ . The fluid-mechanic characteristics of the problem are simplified by assuming that the velocities  $U_g$  and  $U_d$  are constant and that no exchange of momentum takes place between the gas and the liquid. This is essentially an inviscid-flow assumption that permits the solution of the problem in terms of Green's functions and integral equations.

Density variations in the gas phase due to the vaporization process may account at the most by about ten percent of the total density change. Gas density is mainly affected by temperature changes due for example to chemical reactions. Accordingly, neglecting density variations, the equations of conservation of energy and conservation of the fuel species are:

$$U \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial x} - D \frac{\partial^2 T}{\partial x^2} = - \sum_{j=1}^{II} (T - T_d + \frac{L_{eff}}{C_p}) \frac{n}{\rho} \delta(x - x_j) \dot{m} \quad (1a)$$

$$U \frac{\partial Y_F}{\partial z} + v \frac{\partial Y_F}{\partial x} - D \frac{\partial^2 Y_F}{\partial x^2} = \sum_{j=1}^{II} (1 - Y_F) \frac{n}{\rho} \delta(x - x_j) \dot{m} \quad (1b)$$

where  $\alpha$  and  $D$  are the thermal and mass diffusivities respectively,  $n$  is the number of droplets per unit area on the sheets,  $\rho$  is the gas density and  $\delta(x - x_j)$  is the Dirac delta function which indicates that the source terms

are zero everywhere except at the droplet locations.  $L_{eff}$  is an effective heat of vaporization that takes into account the energy used to heat up the droplets. The vaporization rate is given by:

$$\dot{m} = 4\pi R_p D (1 + 0.3 Re^{0.5}) \ln(1 + B_{eff}) \quad (2)$$

where  $B_{eff}$  is an effective Spalding transfer number.

For the liquid phase, an infinite-conductivity model is used in which a uniform but varying droplet temperature is assumed. The equations of conservation of mass and conservation of energy for the droplets are:

$$U \frac{dm}{dz} = - \dot{m} \quad (3a)$$

$$U \frac{dT_l}{dz} = \left( \frac{L_{eff} - L_b}{m C_l} \right) \dot{m}$$

where  $m = 4\pi R_p^3 \rho_l / 3$  is the droplet mass and  $L_b$  is the heat of vaporization of the liquid. Other symbols are defined in the nomenclature.

The boundary conditions are as follows:

At the inlet,  $z = 0$ :  $T = T_1(x)$ ,  $Y_F = Y_{F1}$

$$T_l = T_{l1}, \quad m = m_1 \quad (4)$$

At the boundary planes:

$$\begin{array}{lll} x = 0, & T = T_H, & \frac{\partial Y_F}{\partial x} = 0 \\ x = L, & T = T_C, & \frac{\partial Y_F}{\partial x} = 0 \end{array} \quad (5)$$

Dimensionless variables are selected as:

$$\xi = \frac{z}{r_{\text{vap}} U_1}, \quad r_{\text{vap}} = \frac{m_1}{4\pi R_1 \rho D} = \frac{R_1^2 \rho_1}{3 D \rho}, \quad x = \frac{r}{L} \quad (6a)$$

$$T' = \frac{T - T_{\infty 1}}{T_c - T_{\infty 1}}, \quad T'_1 = \frac{T_1 - T_{\infty 1}}{T_c - T_{\infty 1}}, \quad R' = \frac{R}{R_1} \quad (6b)$$

Introducing these dimensionless variables in eqs. (1) and (3) and neglecting the convective terms in the transverse direction, they become:

$$\frac{\partial T'}{\partial \xi} - P_1 \frac{\partial^2 T'}{\partial x^2} = \sum_{j=1}^{II} -P_2 \left(1 + \frac{1}{B_{\text{eff}}}\right) (T' - T'_1) S_v \delta'(x - x_j) \quad (7a)$$

$$\frac{\partial Y_F}{\partial \xi} - P_1 \frac{\partial^2 Y_F}{\partial x^2} = \sum_{j=1}^{II} P_2 (1 - Y_F) S_v \delta'(x - x_j) \quad (7b)$$

$$\frac{dR'^3}{d\xi} = -S_v \quad (7c)$$

$$\frac{dT'_1}{d\xi} = \frac{C}{C_1} \left( \frac{T' - T'_1}{B_{\text{eff}}} - \frac{1}{P_3} \right) \frac{S_v}{R'^3} \quad (7d)$$

where:

$$S_v = R' C_{\text{Re}} \ln(1 + B_{\text{eff}})$$

$$C_{\text{Re}} = 1 + 0.3 (\text{Re})^{1/2} = 1 + 0.3 P_4 R'^{1/2}$$

$$B_{\text{eff}} = \frac{C (T' - T'_1)}{P L_{\text{eff}}} = \frac{Y_{\text{FS}} - Y_F}{1 - Y_{\text{FS}}}$$



The boundary conditions become:

$$\text{At } \xi = 0: \quad T' = T'_1(\chi), \quad Y_F = Y_{F1}, \quad T'_L = 0, \quad R' = 1 \quad (8a)$$

$$\text{At } \chi = 0: \quad T' = T'_H, \quad \frac{\partial Y_F}{\partial \chi} = 0 \quad (8b)$$

$$\text{At } \chi = 1: \quad T' = 1, \quad \frac{\partial Y_F}{\partial \chi} = 0 \quad (8c)$$

Equations (7) contain the following dimensionless parameters which characterize the problem:

$$\begin{aligned} P_1 &= \frac{R_1^2 \rho_L U_L}{3L^2 \rho_g U_g} & P_2 &= \frac{nm_1 U_L}{L \rho U_g} \\ P_3 &= B_1 = \frac{C(T_c - T_{L1})}{L_b} & P_4 &= Re_1 = \frac{2R_1(U_g - U_L)}{\nu} \end{aligned} \quad (9)$$

Other parameters are: the relative positions of the droplet sheets, the ratio of the specific heats as well as other dimensionless quantities appearing through the equilibrium equations:

$$\frac{1}{Y_{FS}} = 1 + \frac{M_g}{M_L} \left( \frac{1}{X_{FS}} - 1 \right) \quad (10a)$$

$$X_{FS} = \frac{P_{atm}}{P} \exp \left[ \frac{L_b M_L}{R^o} \left( \frac{1}{T_{b \text{ atm}}} - \frac{1}{T_L} \right) \right] \quad (10b)$$

We concentrate our attention on the effect of the parameters in (9) as these are very influential in the behavior of the gas and liquid phases and in particular, in the development of the fuel concentration field in the gas

phase. The first parameter,  $P_1$ , can be viewed as a ratio between a characteristic vaporization time and a characteristic gas-diffusion time. If  $P_1$  were small, vaporization of the droplets would take place in a time which is shorter than the time required by the gas diffusion mechanism to distribute the fuel over the cross sectional area. In the limit  $P_1 \rightarrow 0$  there will be two infinitely narrow bands of fuel vapor at the droplet locations and essentially no fuel anywhere else in the gas. As  $P_1$  increases, diffusion tends to smooth out the fuel concentration profiles. The second parameter,  $P_2$ , represents the ratio of the rates of incoming liquid mass per sheet and incoming gas mass at the channel inlet.  $P_2$  is generally less than unity and as it increases, the result is an increase in the local and total amount of fuel in the gas phase. The third parameter,  $P_3$ , corresponds to the Spalding transfer number based on the reference temperatures  $T_c$  and  $T_{g1}$  and on the fuel latent heat of vaporization  $L_p$ . The last parameter,  $P_4$ , is the Reynolds number based on initial droplet diameter and relative velocity. Solution of Eqs. (7) subject to the conditions given by (8) is obtained with the aid of the Green's functions<sup>8</sup>  $G_T(\xi, \chi | \xi', \chi')$  and  $G_F(\xi, \chi | \xi', \chi')$  which satisfy

$$\frac{\partial G_T}{\partial \xi} - P_1 \frac{\partial^2 G_T}{\partial \chi^2} = \delta'(\xi - \xi') \delta'(\chi - \chi')$$

$$G_T = 0 \text{ at } \chi = 0 \text{ and } \chi = 1$$

$$G_T = 0 \text{ for } \xi < \xi'$$

(11a)

$$\frac{\partial G_F}{\partial \xi} - P_1 \frac{\partial^2 G_F}{\partial \chi^2} = \delta'(\xi - \xi') \delta'(\chi - \chi')$$

$$\partial G_F / \partial \chi = 0 \text{ at } \chi = 0 \text{ and } \chi = 1$$

$$G_F = 0 \text{ for } \xi < \xi' \quad (11b)$$

Equations (11) exactly correspond to the diffusion equation with an instantaneous point source located at  $(\xi', \chi')$  for Dirichlet and Neumann conditions, respectively<sup>9</sup>. The solutions are:

$$G_T = \sum_{n=1}^{\infty} 2 \sin(n\pi\chi) \sin(n\pi\chi') e^{-\gamma_n(\xi-\xi')} \quad (12a)$$

$$G_F = 1 + \sum_{n=1}^{\infty} 2 \cos(n\pi\chi) \cos(n\pi\chi') e^{-\gamma_n(\xi-\xi')} \quad (12b)$$

$$\gamma_n = P_1 n^2 \pi^2 \quad (13)$$

The solution of eqs. (7a) and (7b) is now expressed in terms of the Green's functions  $G_T$  and  $G_F$  in the form of integral equations containing the influence of the source terms and boundary and inlet conditions:

$$\begin{aligned} T'(\xi, \chi) = & \int_0^1 (T' G_T)|_{\xi'=0} d\chi' + P_1 \int_0^{\xi} T' H \left( \frac{\partial G_T}{\partial \chi'} \right) |_{\chi'=0} d\xi' \\ & - P_1 \int_0^{\xi} \left( \frac{\partial G_T}{\partial \chi'} \right) |_{\chi'=1} d\xi' + \sum_{j=1}^{II} \int_0^{\xi} (S_T G_T) |_{\chi'=\chi_j} d\xi' \end{aligned} \quad (14a)$$

$$Y_F(\xi, \chi) = \int_0^1 (Y_F G_F) |_{\xi'=0} d\chi' + \sum_{j=1}^{II} \int_0^{\xi} (S_F G_F) |_{\chi'=\chi_j} d\xi' \quad (14b)$$

where the source terms  $S_T$  and  $S_F$  are given by:

$$S_T = -P_2 (T' - T'_L) \left( 1 + \frac{1}{B_{eff}} \right) S_v \quad (15a)$$

$$S_F = P_2 (1 - Y_F) S_v \quad (15b)$$

Solution of equations (14a) and (14b) for the gas temperature and fuel concentration, and of equations (7c) and (7d) for the droplet radius and temperature requires an iterative scheme because the unknown functions appear in the integrals of the source terms. A more efficient numerical solution can be achieved by differentiating equations (14a) and (14b) with respect to  $\xi$ , a procedure that transforms them into ordinary differential equations which can be readily solved with a Runge-Kutta method. We note the series-character of the Green's functions, and therefore of the kernels in the integral source terms, and let the functions  $T'$  and  $Y_F$  be composed of an infinite number of functions plus some known functions related to the inlet and boundary conditions:

$$T' = f_T + \sum_{n=1}^{\infty} 2T'_n \sin(n\pi\chi) \quad (16a)$$

$$Y_F = f_F + Y_{fo} + \sum_{n=1}^{\infty} 2Y_{Fn} \cos(n\pi\chi) \quad (16b)$$

Here,  $f_T$  corresponds to the first three terms of equation (14a) while  $f_F$  corresponds to the first term of equation (14b).

Upon differentiating equations (14a) and (14b) with respect to  $\xi$  and comparing terms in the infinite series of equations (12) and (16), and after some manipulation one obtains:

$$\frac{dT'_n}{d\xi} + \gamma_n T'_n = \sum_{j=1}^{II} (S_T \sin(n\pi\chi')) \Big|_{\chi'=\chi_j} \quad n = 1, 2, 3, \dots \quad (17a)$$

$$\frac{dY_{Fn}}{d\xi} + \gamma_n Y_{Fn} = \sum_{j=1}^{II} (S_F \cos(n\pi x_j')) \big|_{x'=\chi_j} \quad n = 0, 1, 2, \dots \quad (17b)$$

subject to the conditions:

$$T'_n = Y_{Fn} = 0 \text{ at } \xi = 0 \quad (17c)$$

Equations (17) are solved together with equations (7c) and (7d) using a Runge-Kutta-Gill 4th order scheme up to the value of  $n$  for which certain convergence criteria are met.

#### Results

The results of the analysis will be presented in the following manner. First, we show and discuss the results of a characteristic case, including droplet temperature and size histories as well as the development of the gas-phase temperature and fuel-concentration profiles. We continue the presentation of the results with an analysis of the effects of the selected parameters,  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ .

The standard case corresponds to liquid n-heptane droplets at 300 K and 150  $\mu\text{m}$  in diameter injected with a velocity of 1 m/s in an air stream with a linear temperature profile which goes from 2050 K on the left boundary to 1000 K on the right boundary and a velocity of 5 m/s. The dimensionless parameters have the following values:  $P_1 = 0.1$ ,  $P_2 = 0.1$ ,  $P_3 = 2$ ,  $P_4 = 12$ . The droplet sheets are located at one third and two thirds of the distance from the left wall to the right wall.

Figures 1 and 2 show the droplet-temperature and droplet-radius histories on the two droplet sheets. The first sheet, I, is closer to the hot boundary and therefore these droplets heat up and vaporize faster. There is a

transient heating period of roughly 33% of the droplet's lifetime during which almost 40% of the droplet is vaporized and after which an almost constant droplet temperature is established. This temperature is appreciably lower than the saturation temperature at the prevailing pressure ( $T'_{\text{sat}} = 0.22$ ). Note that with less volatile fuels, the transient behavior would be even more pronounced.

Figure 3 shows the development of the fuel mass fraction profiles in the gas phase. The two maxima correspond to the location of the droplet sheets. The sheet located at  $\chi = 1/3$  is exposed to a higher gas temperature and therefore vaporizes faster and produces more fuel vapor than the sheet located at  $\chi = 2/3$ . However, after the droplets on the first sheet are completely vaporized at  $\xi = 0.725$  and due to the effect of gas diffusion, the fuel concentration at the location of the second sheet is eventually higher than anywhere else. Eventually, as  $\xi$  goes to infinity, a uniform profile results.

The gas-phase temperature is given in Figure 4 at three different axial locations. At the inlet,  $\xi = 0$ , the temperature profile is linear but due to heat transfer to the droplets, two energy sinks are developed at the droplet-sheet locations which produce the two temperature depressions near  $\chi = 1/3$  and  $\chi = 2/3$  ( $\xi = 0.2$ ). Eventually ( $\xi = 1$ ) the two depressions are smoothed out by the gas diffusion and as  $\xi \rightarrow \infty$ , the temperature profile recovers its linear shape.

#### Effect of $P_1$

Figures 5 and 6 correspond to a four-fold increase of  $P_1$  ( $P_1 = 0.4$ ) while other parameters are kept unchanged.

Increasing  $P_1$  results in a reduction of the time required by the gas-diffusion mechanism to smooth out the differences created by the heat losses

and mass gains occurring at the droplet-sheet locations. This results in less pronounced maxima in the fuel mass fraction profiles and a smoother distribution of fuel vapor over the cross sectional area as shown in Figure 5. The droplet-size histories on the two droplet sheets for this case are shown in Figure 6. Since the droplets are now exposed to a locally higher temperature, the vaporization times are reduced by approximately 7% with respect to those of Figure 2.

#### Effect of $P_2$

The influence of  $P_2$  is discussed with the aid of Figure 7. This figure corresponds to a four-fold reduction of  $P_2$  ( $P_2 = 0.025$ ) while keeping all other parameters as in the base case. As explained in the Analysis section,  $P_2$  is the ratio of the incoming liquid and gas mass flows. Figure 7 shows that a reduction in  $P_2$  by a factor of 4 nearly directly correlates with a reduction by the same factor of the fuel vapor present in the gas phase. Another consequence not presented here is a reduction in the droplet lifetimes of roughly 10% with respect to the base case.

#### Effect of $P_3$

A reduction by a factor of two of the transfer number  $P_3$  produces a marked increase in the droplet lifetime (45% in this case) as indicated by Figure 8 and a reduction of the final droplet temperature. This is a consequence of an increase in the ratio of the heat required to vaporize the droplets to the heat available in the gas-phase. The development of the fuel mass fraction profile in the gas phase takes longer here than in the base case as can be seen in Figure 9.

#### Effect of $P_4$

The Reynolds number appears in the problem through the relative-velocity correction in the mass vaporization rate of Eqn (2). With the assumptions of constant relative velocity between the gas and liquid phases and constant physical properties, the Reynolds number,  $P_4$ , varies throughout the flow only due to variations in droplet size. Decreasing the initial Reynolds number reduces the mass vaporization rate and therefore increases the droplet lifetime. Figures 2 and 10 show an increase of droplet lifetimes of approximately 25% when the Reynolds number is reduced from 12 to 4. Accordingly, comparing Figures 3 and 11, it is observed that the amount of fuel present in the gas phase in the early stages of the process ( $\xi = 0.2$ ) is about 30% less in the lower Reynolds number case. Note, however, that towards the end of the droplet lifetimes ( $\xi = 0.8$ ) that difference has almost disappeared.

#### Conclusions

A simplified but qualitatively reasonable model of heat transfer and rapid vaporization in a two-dimensional spray has been presented. Based on the assumption of uniform flow velocities and the neglect of viscous exchange of momentum between the gas and liquid phases, an analytically tractable system of partial differential equations has been obtained which has been further transformed to a system of integral equations with the use of Green's functions. While the model introduces considerable simplification by removing the nonlinearities from the differential operators, it retains the complexity and nonlinearity of the source terms, making it possible to consider the inclusion in the model of the chemical-reaction terms. The analysis



identifies several dimensionless parameters that characterize the problem and the influence of four of these is presented in detail, namely: a characteristic vaporization-diffusion time ratio, a liquid-to-gas mass flow ratio, a Spalding transfer number, and a Reynolds number.

The model offers some quantitatively crude but qualitatively very informative results of the complex mechanism of heat and mass transfer occurring prior to the ignition of two parallel fuel droplet streams, and in particular it serves as a base for a simplified model of ignition of a droplet stream by another droplet stream, a topic that is presently under investigation.

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# Nomenclature

|                   |  |
|-------------------|--|
| $B_1$             | reference transfer number defined in (9)             |
| $B_{eff}$         | effective transfer number                            |
| $C_l$             | liquid specific heat                                 |
| $C_p$             | gas constant-pressure specific heat                  |
| $C_{Re}$          | convective correction, $C_{Re} = 1 + 0.3 (Re)^{1/2}$ |
| $D$               | mass diffusion coefficient                           |
| $f_T, f_T$        | functions described in the context of Eqns (16a,b)   |
| $G_T, G_T$        | Green's functions described by Eqns (12a,b)          |
| $L$               | channel width  |
| $L_{eff}$         | effective heat of vaporization                       |
| $L_b$             | latent heat of vaporization                          |
| $m$               | droplet mass   |
| $\dot{m}$         | mass vaporization rate given in (2)                  |
| $M_g$             | gas molecular weight                                 |
| $M_l$             | liquid molecular weight                              |
| $n$               | droplet-sheet density; series index                  |
| $p$               | pressure   |
| $p_{atm}$         | normal atmospheric pressure                          |
| $P_1, \dots, P_4$ | parameters defined in (9)                            |
| $R$               | droplet radius                                       |
| $R'$              | normalized droplet radius, $R' = R/R_1$              |
| $R_1$             | initial droplet radius                               |
| $R^0$             | universal gas constant                               |
| $Re$              | Reynolds number, $Re = \frac{2R U_g - U_l }{\nu}$    |
| $S_T, S_T$        | source terms described by Eqns (15a,b)               |
| $T$               | gas temperature                                      |

|          |   |
|----------|---|
| $T'$     | normalized gas temperature, $T' = (T - T_{i1}) / (T_c - T_{i1})$        |
| $T_i$    | inlet gas temperature   |
| $T_c$    | cold wall temperature   |
| $T_H$    | hot wall temperature  |
| $T_l$    | liquid temperature  |
| $T'_l$   | normalized liquid temperature, $T'_l = (T_l - T_{i1}) / (T_c - T_{i1})$ |
| $T'_n$   | n-th component of temperature series (Eq 16a)                           |
| $U_g$    | liquid velocity   |
| $x$      | transverse coordinate   |
| $Y_f$    | fuel mass fraction  |
| $Y_{fs}$ | fuel mass fraction at droplet surface                                   |
| $Y_{fn}$ | n-th component of fuel mass fraction series (Eq 16b)                    |
| $z$      | streamwise coordinate   |

#### Greek symbols

|              |   |
|--------------|---|
| $\alpha$     | thermal diffusivity   |
| $\gamma_n$   | parameter defined in (13)   |
| $\delta$     | Dirac delta function $\delta = \delta(x - x_j)$                           |
| $\delta'$    | dimensionless Dirac delta function $\delta' = \delta'(x - x_j) = L\delta$ |
| $\nu$        | kinematic viscosity   |
| $\xi$        | dimensionless streamwise coordinate (Eqn 6a)                              |
| $\rho$       | gas density   |
| $\rho_l$     | liquid density  |
| $\tau_{vap}$ | characteristic vaporization time defined in (6a)                          |
| $\chi$       | normalized transverse coordinate, $\chi = x/L$                            |

### Subscripts

|       |                                 |
|-------|---------------------------------|
| F     | fuel                            |
| g     | gas                             |
| i     | inlet, initial                  |
| j     | droplet sheet, $j = I, II$      |
| l     | liquid                          |
| n     | series index                    |
| T     | gas temperature                 |
| I, II | first and second droplet sheets |

## References

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### Figure Captions

1. Normalized droplet temperature histories for the base case. I: Droplet sheet near hot wall, II: Droplet sheet near cold wall
2. Normalized droplet size histories for the base case. I: Droplet sheet near hot wall, II: Droplet sheet near cold wall
3. Development of the gas-phase fuel mass fraction profiles for the base case
4. Development of the gas-phase temperature profiles for the base case
5. Development of the gas-phase fuel mass fraction profiles for the case of  $P_1 = 0.4$
6. Normalized droplet size histories for the case of  $P_1 = 0.4$
7. Development of the gas-phase fuel mass fraction profiles for the case of  $P_2 = 0.025$
8. Normalized droplet size histories for the case of  $P_3 = 1$
9. Development of the gas-phase fuel mass fraction profiles for the case of  $P_3 = 1$
10. Normalized droplet size histories for the case of  $P_4 = 4$
11. Development of the gas-phase fuel mass fraction profiles for the case of  $P_4 = 4$

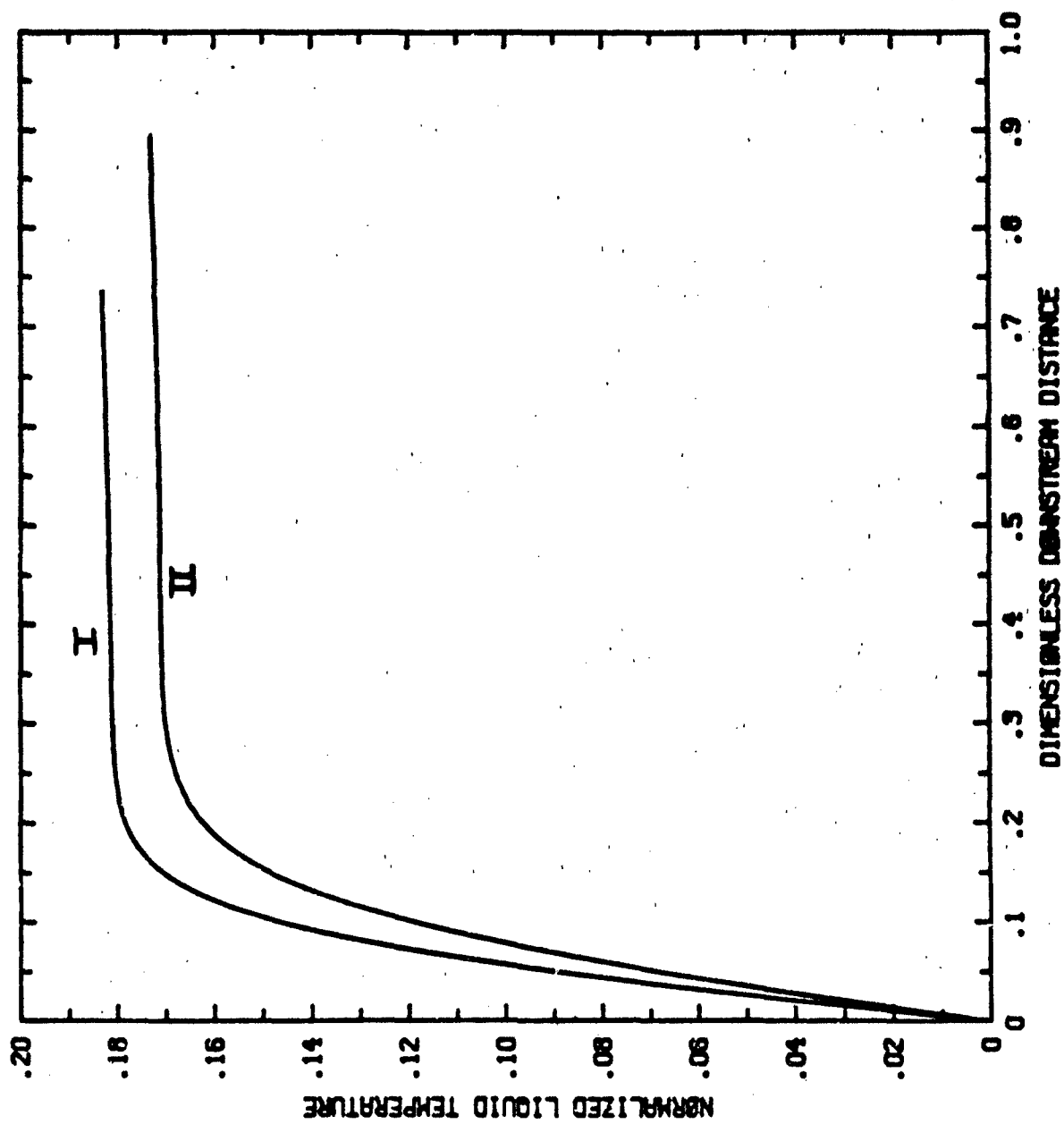


Fig. 1

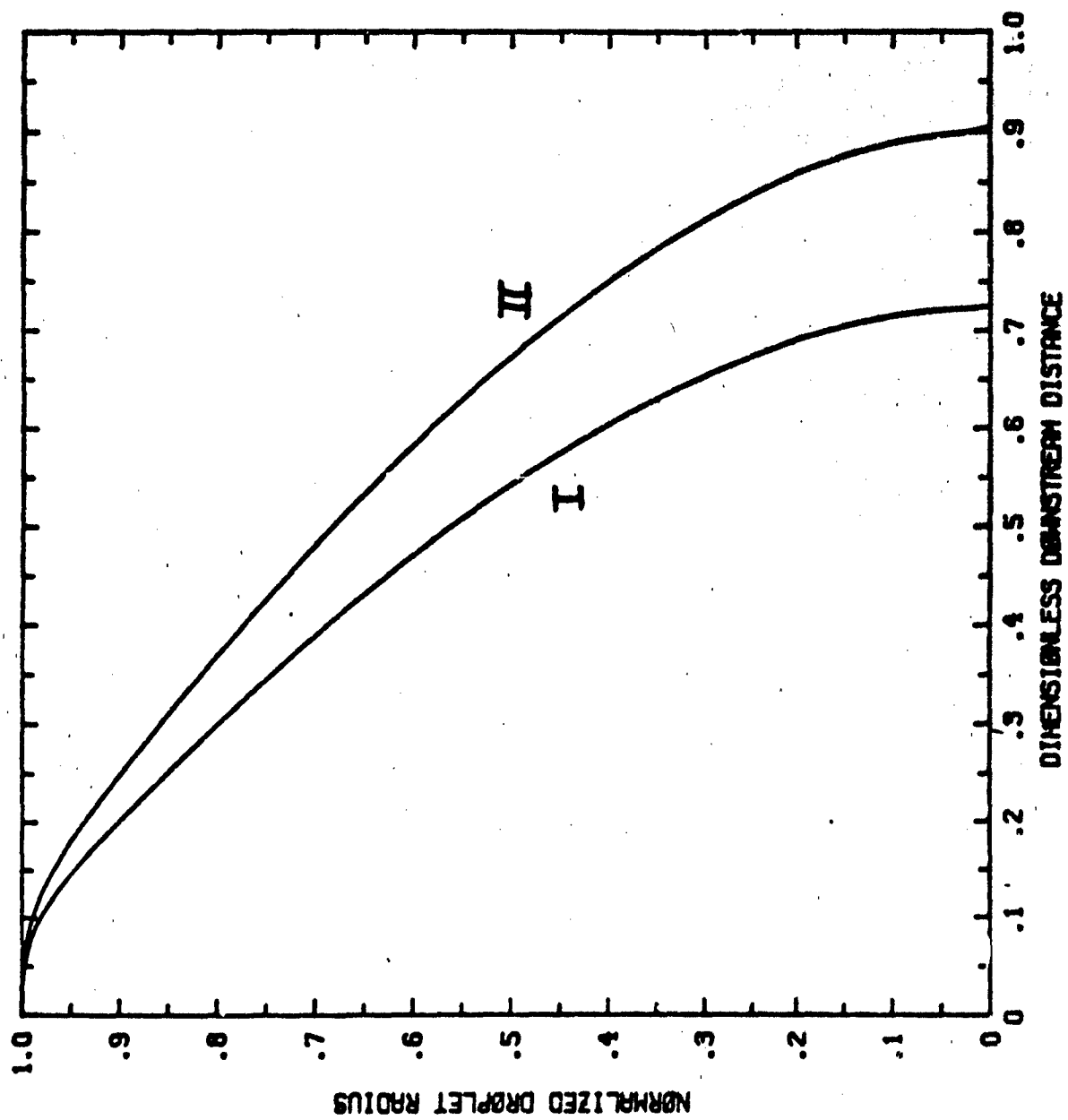


Fig. 2



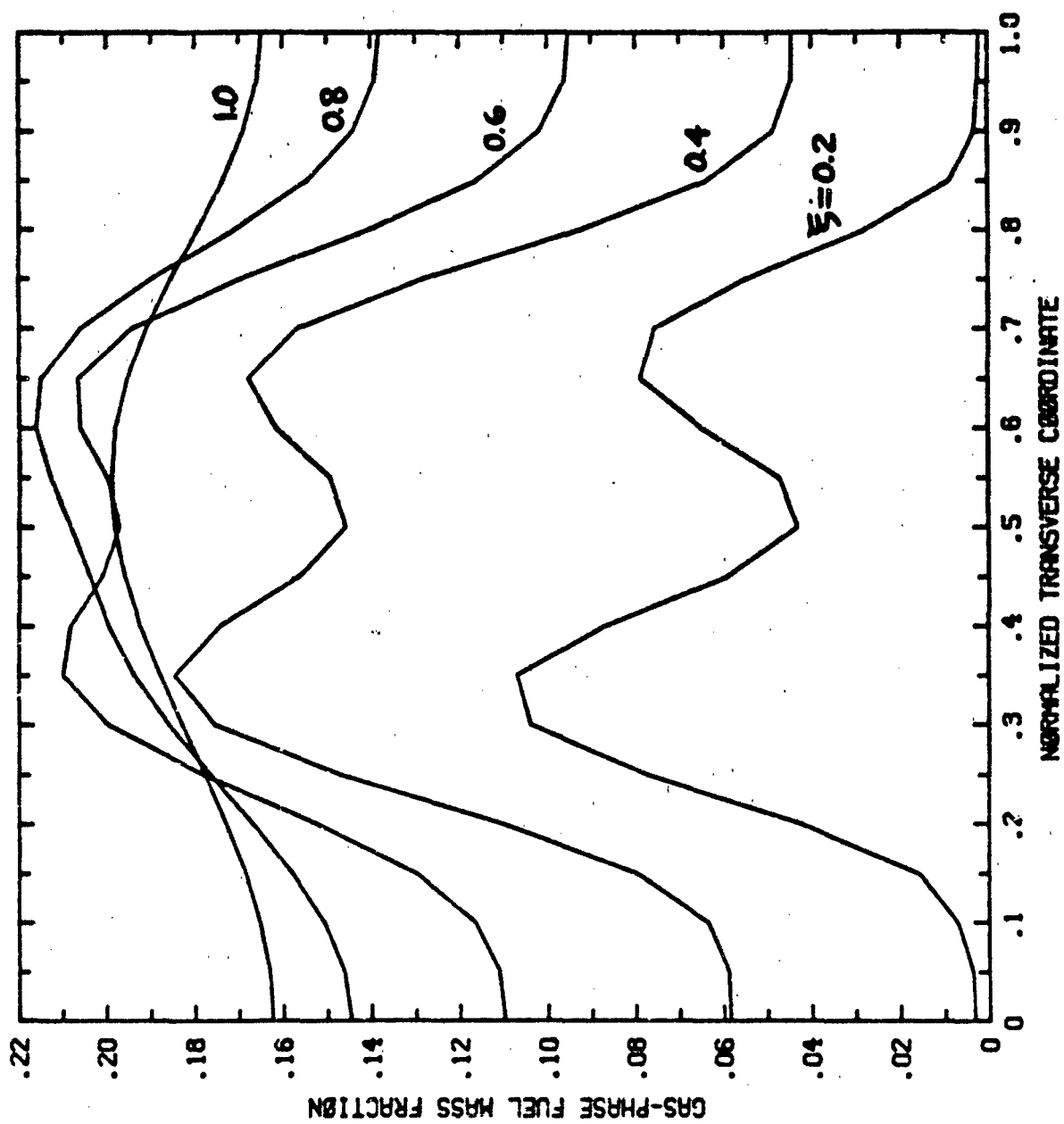


Fig. 3

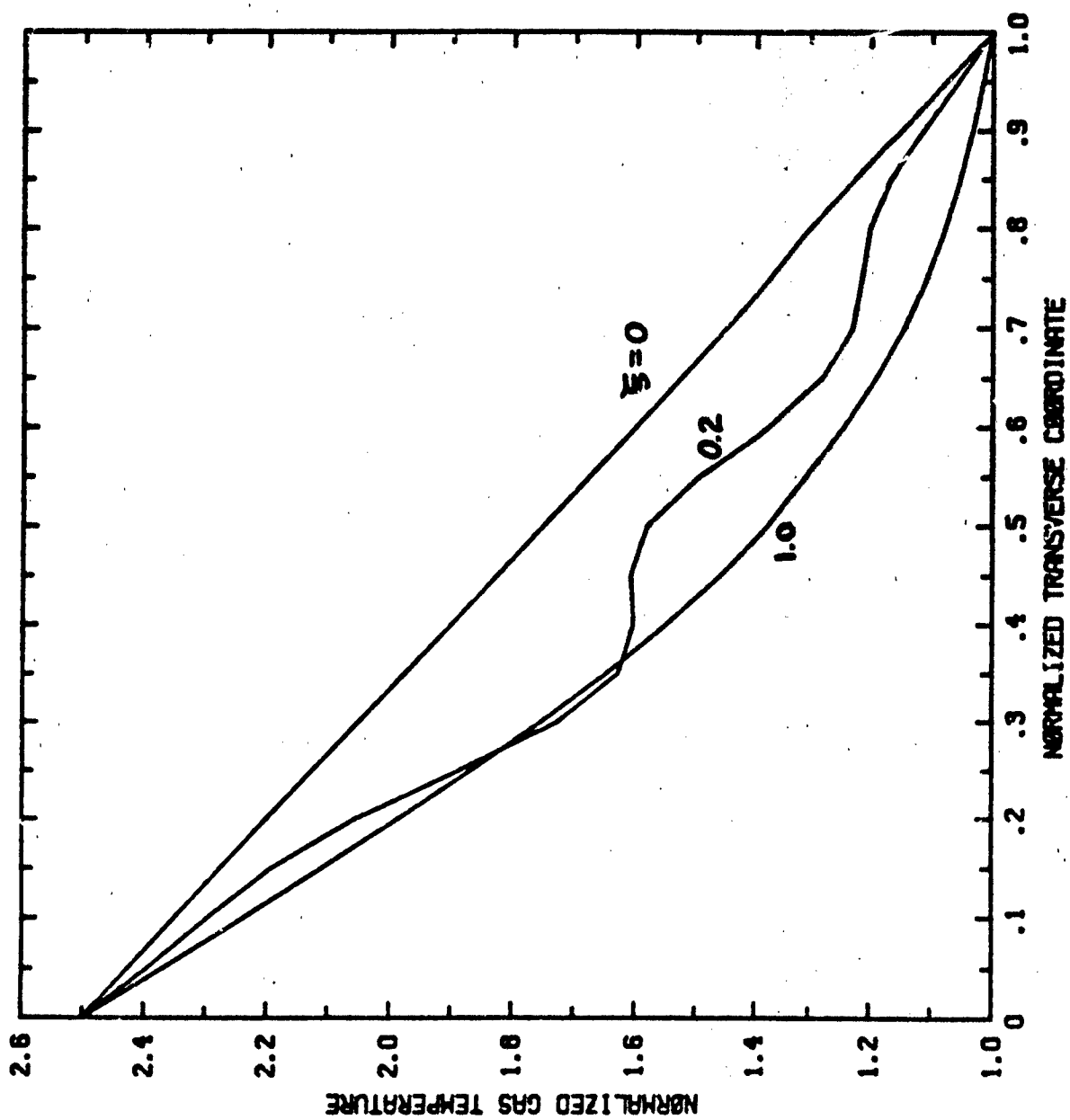


Fig. 4

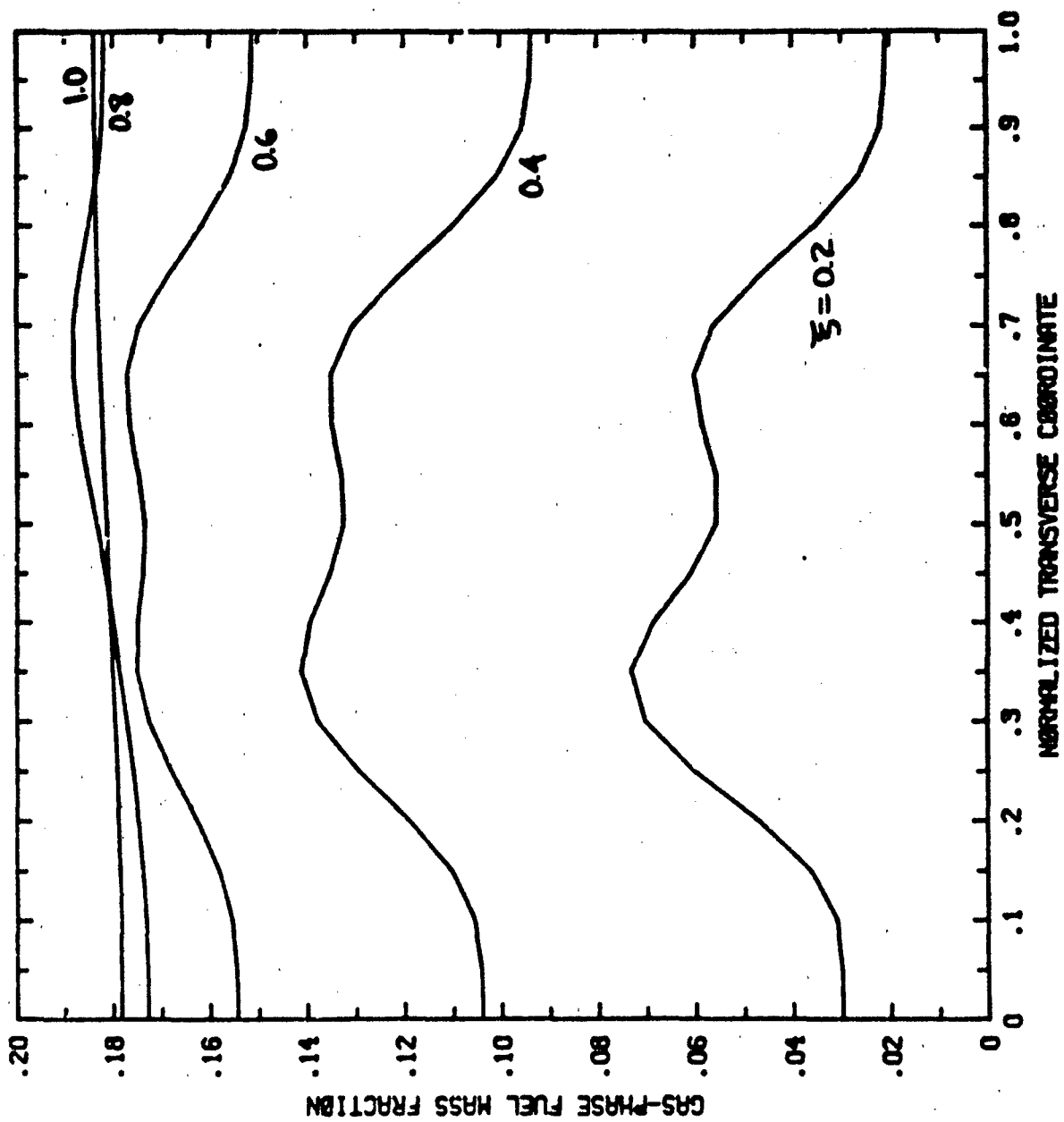


Fig. 5

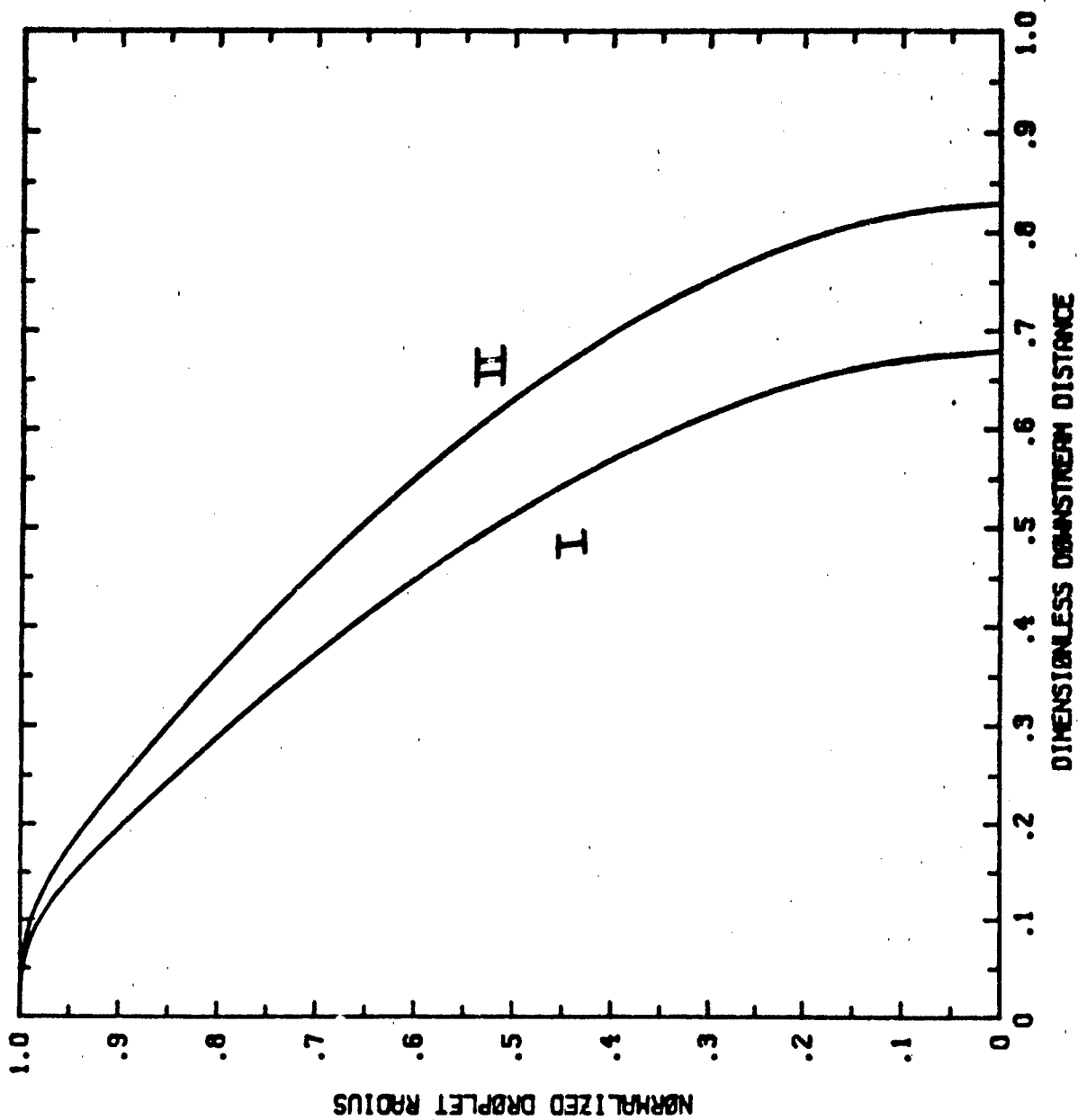


Fig. 6

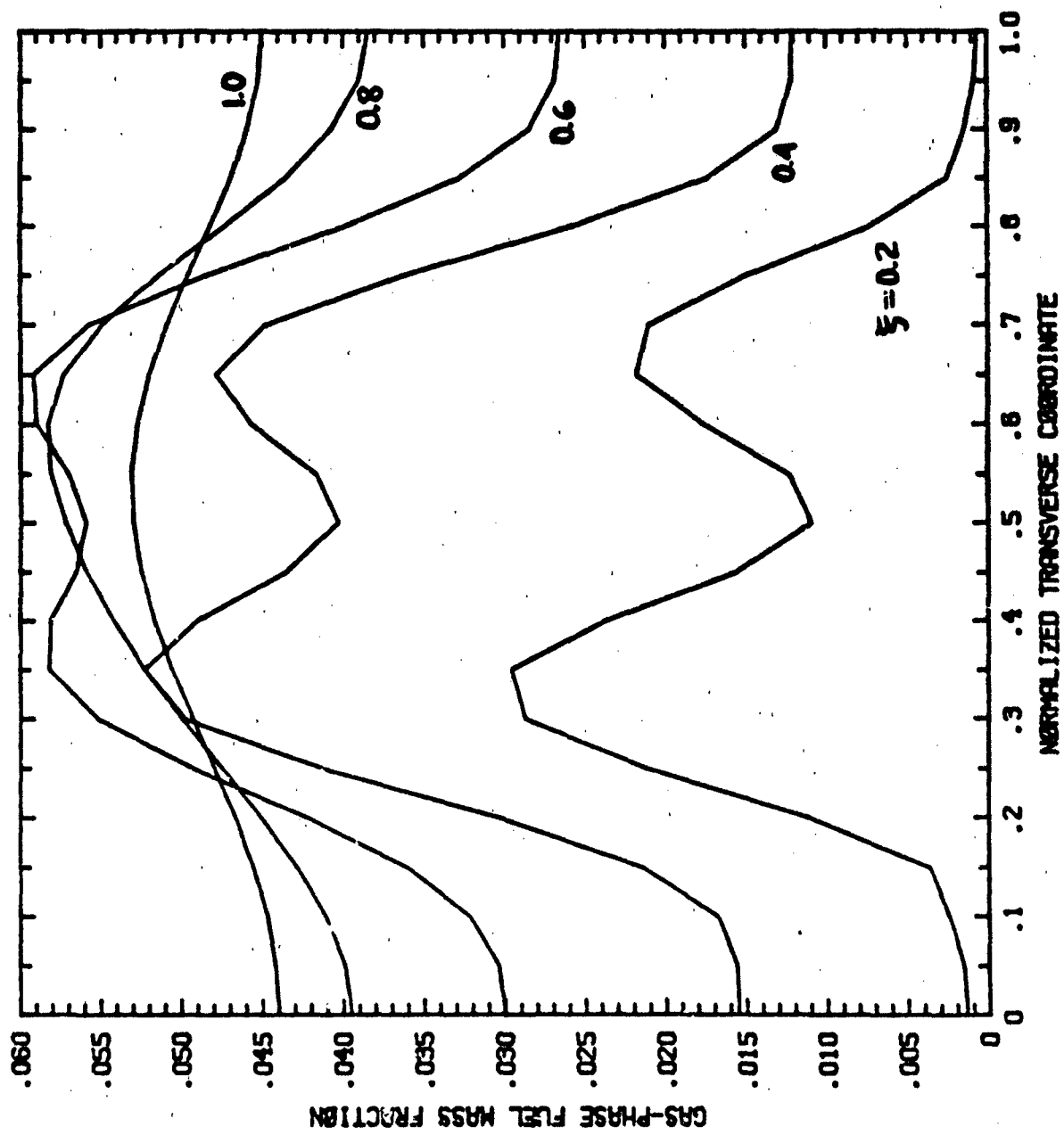


Fig. 7

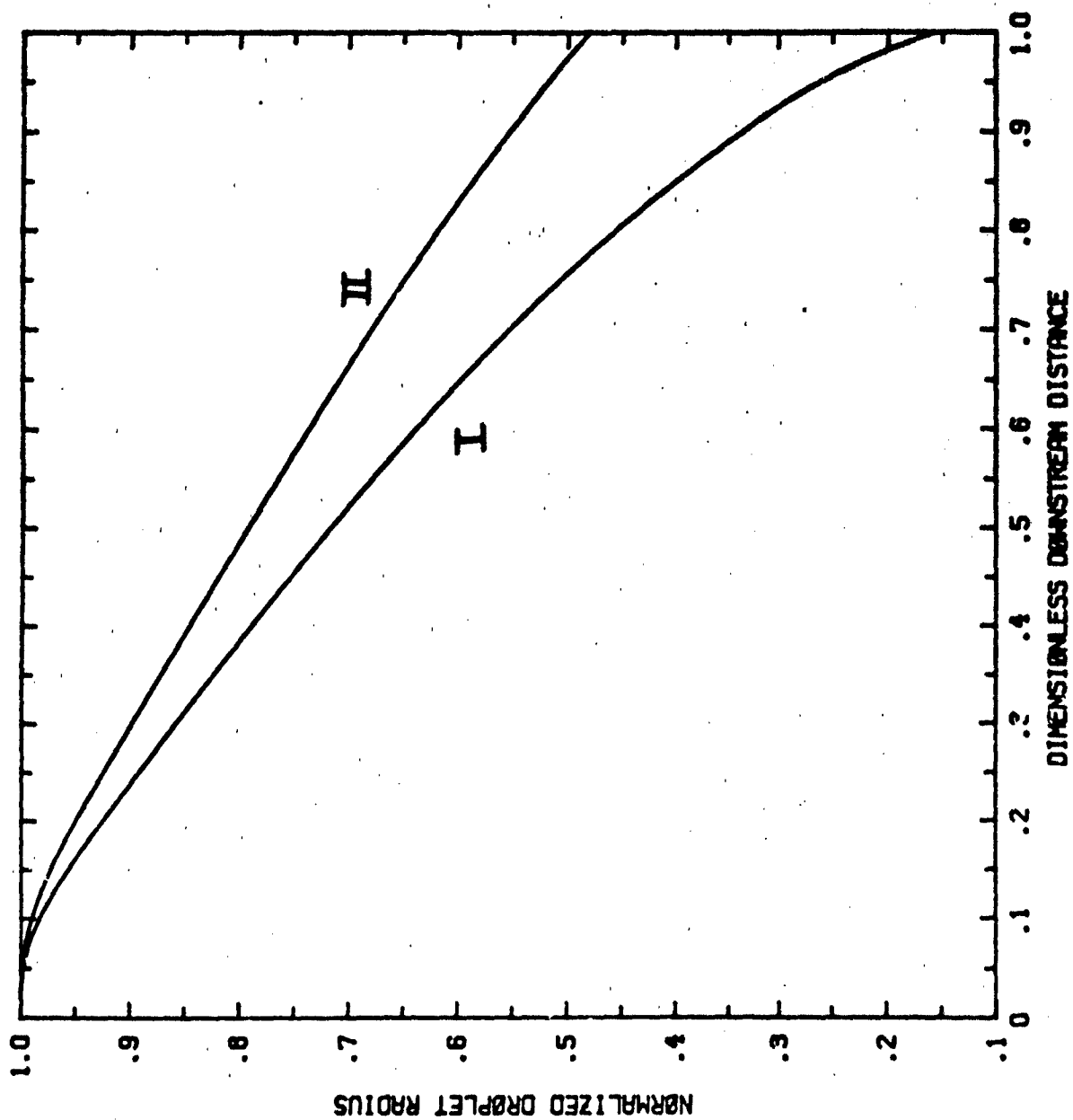


Fig. 8

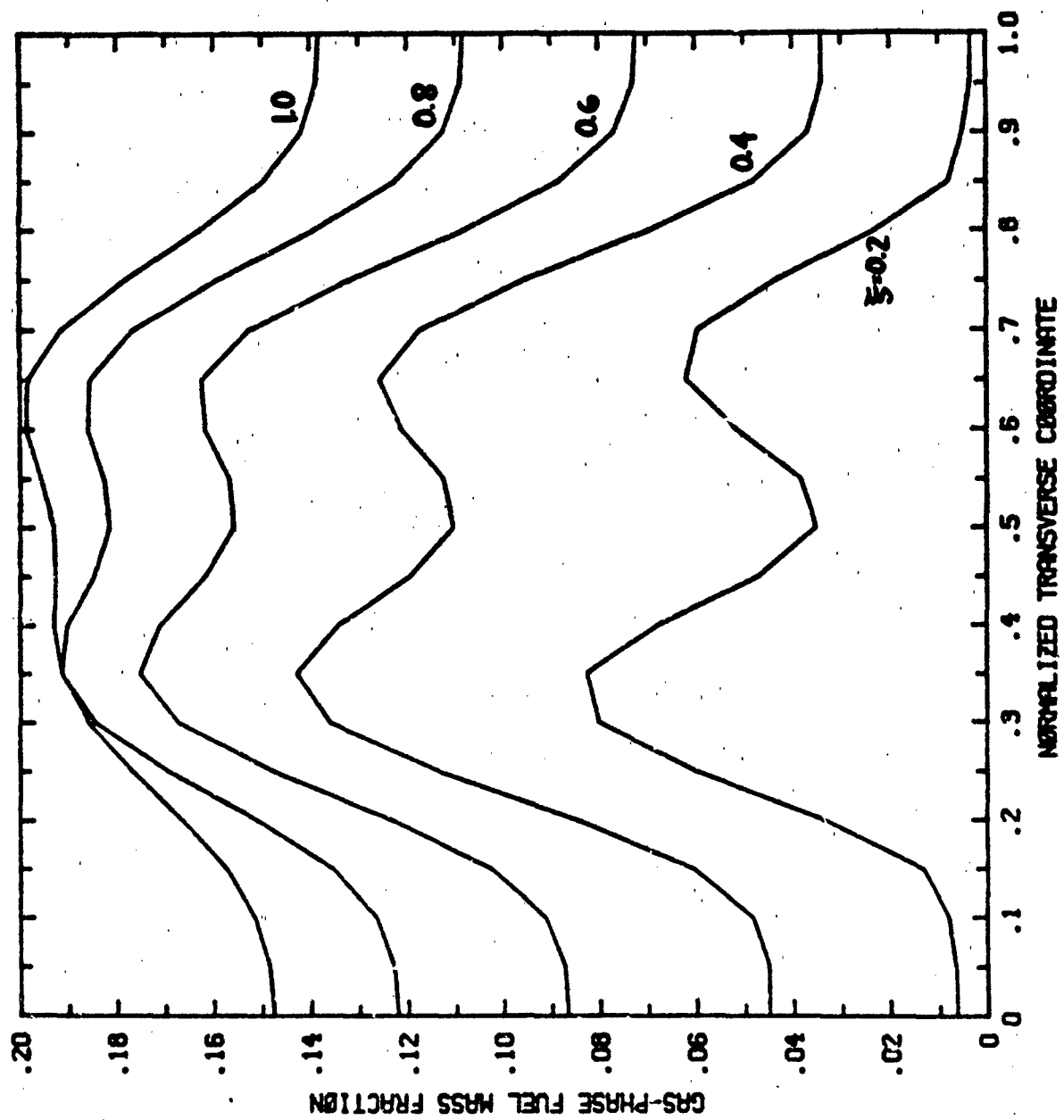


FIG. 9

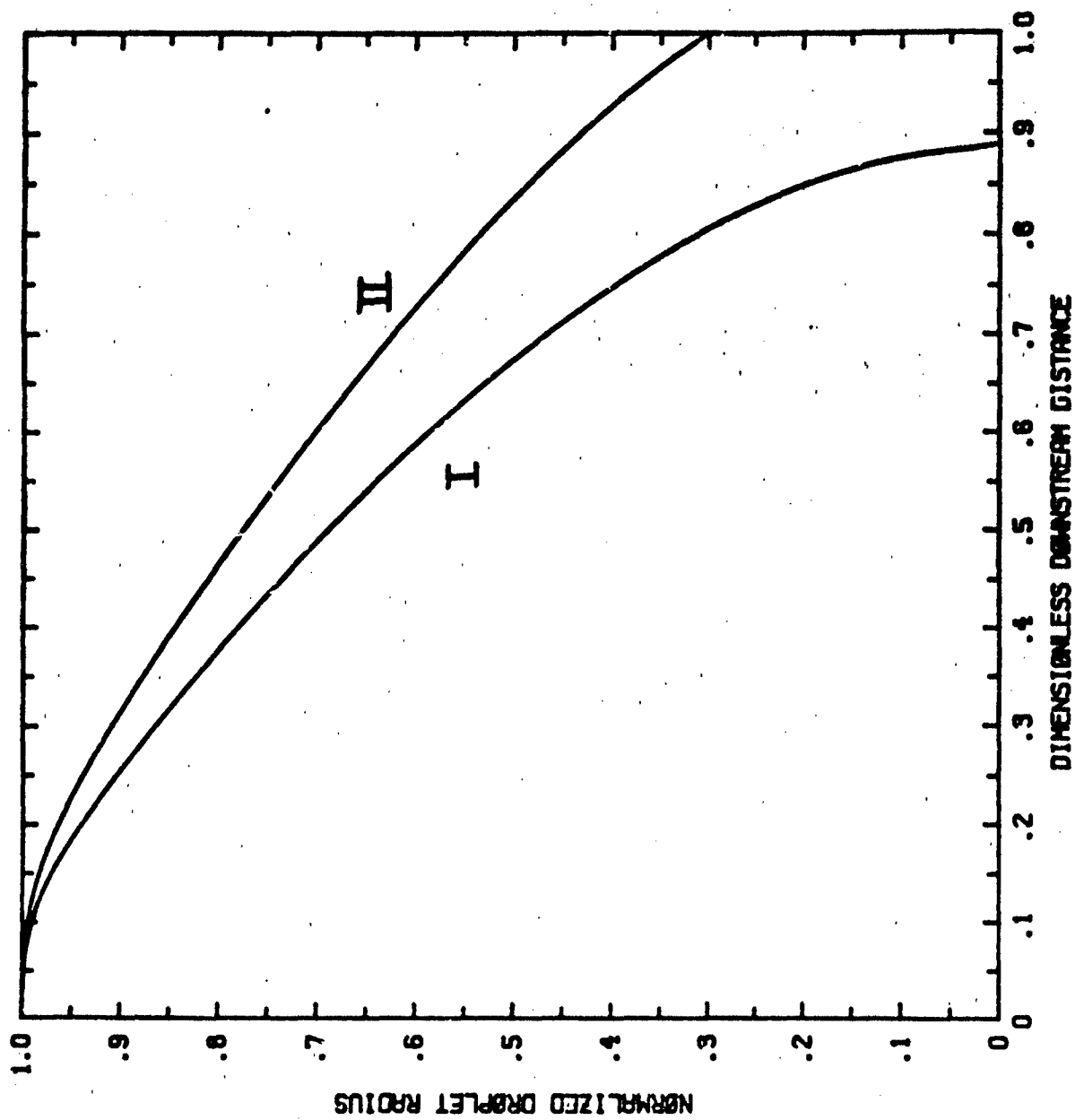


Fig. 10



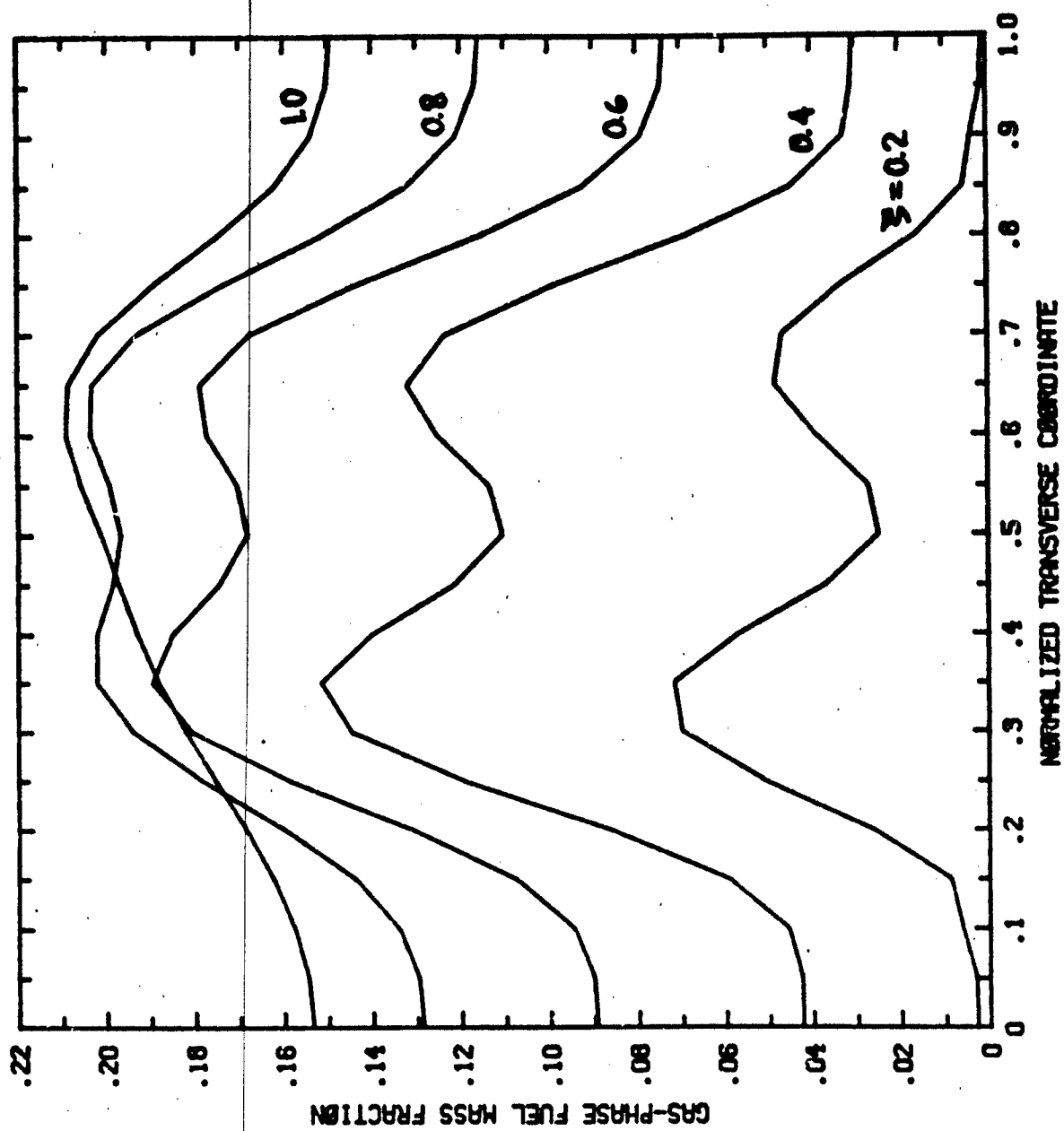


Fig. 11